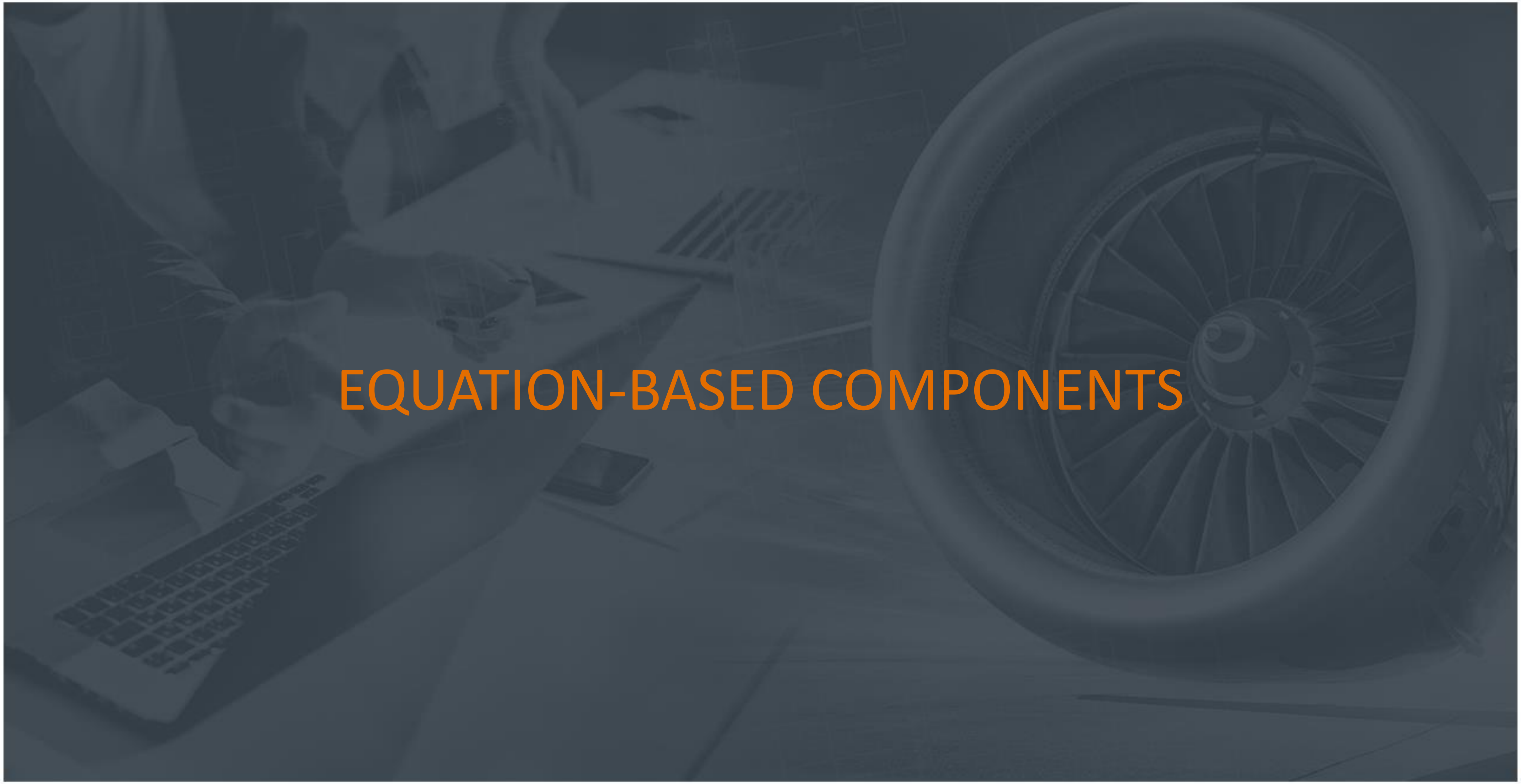
EQUATION-BASED MODELING

Lecture 2.2

Modelon

OVERVIEW

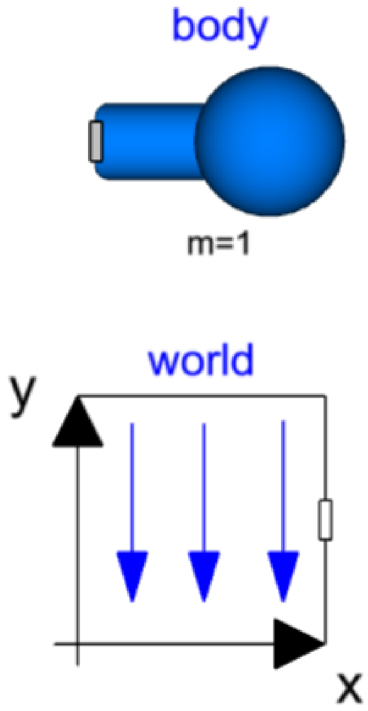
- Equation-based components
- State selection in a dynamic system
- Initialization



EQUATION-BASED COMPONENTS

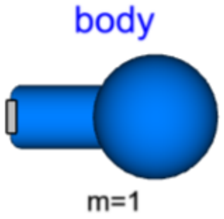
EQUATION-BASED COMPONENTS

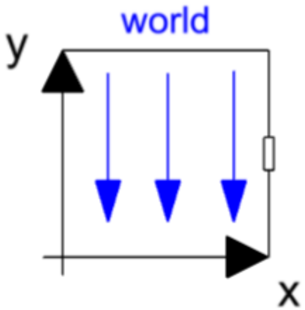
- Consider a physical system, like a mass in a gravity field, there are many different types of analysis to do; for example:
  - Given an initial speed and altitude, when will the mass return to earth?
  - What external force is required to reach a desired altitude profile?
- Independent of the type of analysis you would like to do on this system, the basic textbook equations will always remain the same
- Equation-based modeling is acausal
- Equation-based modeling can be used to create reusable code!

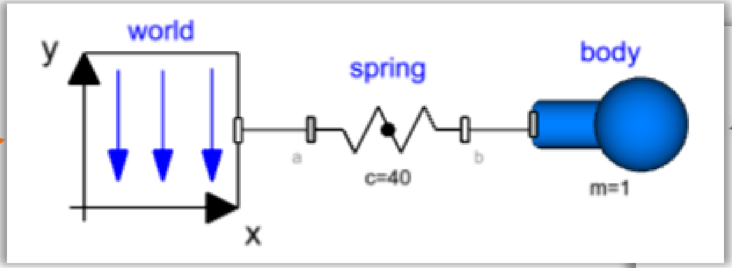


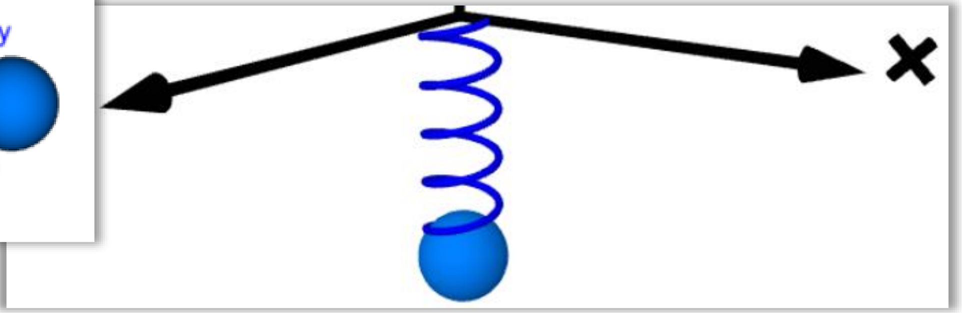
BOUNDARY CONDITIONS

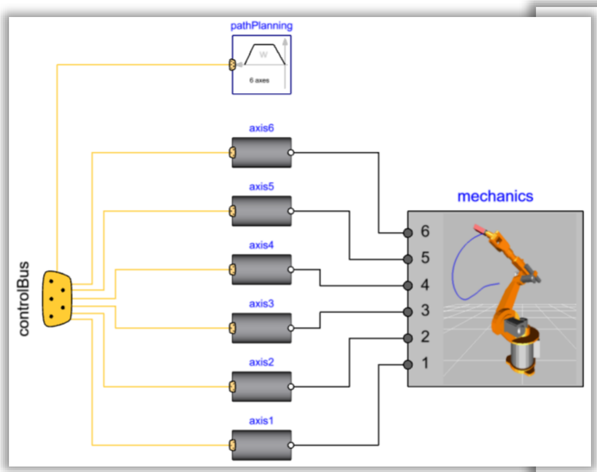
- Equation-based modeling is all about separating the physical and mathematical principles and laws from the boundary conditions of a specific case.

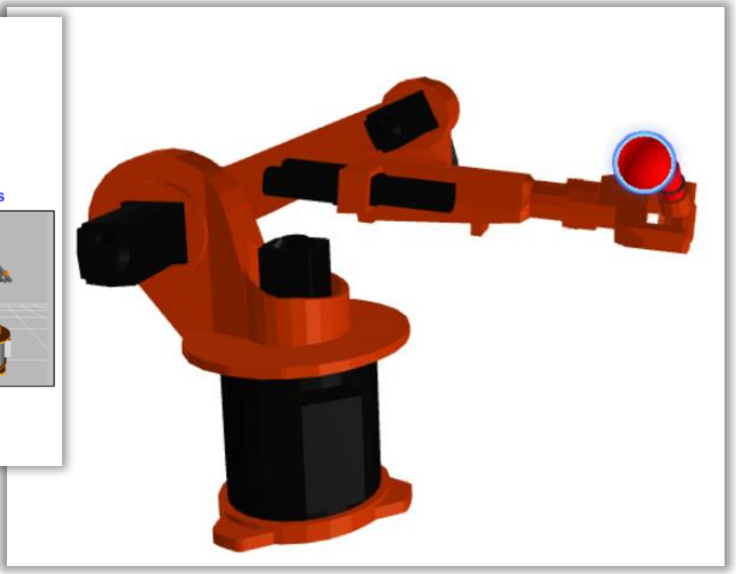






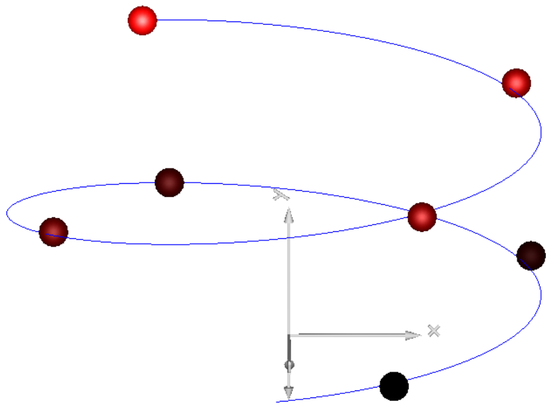






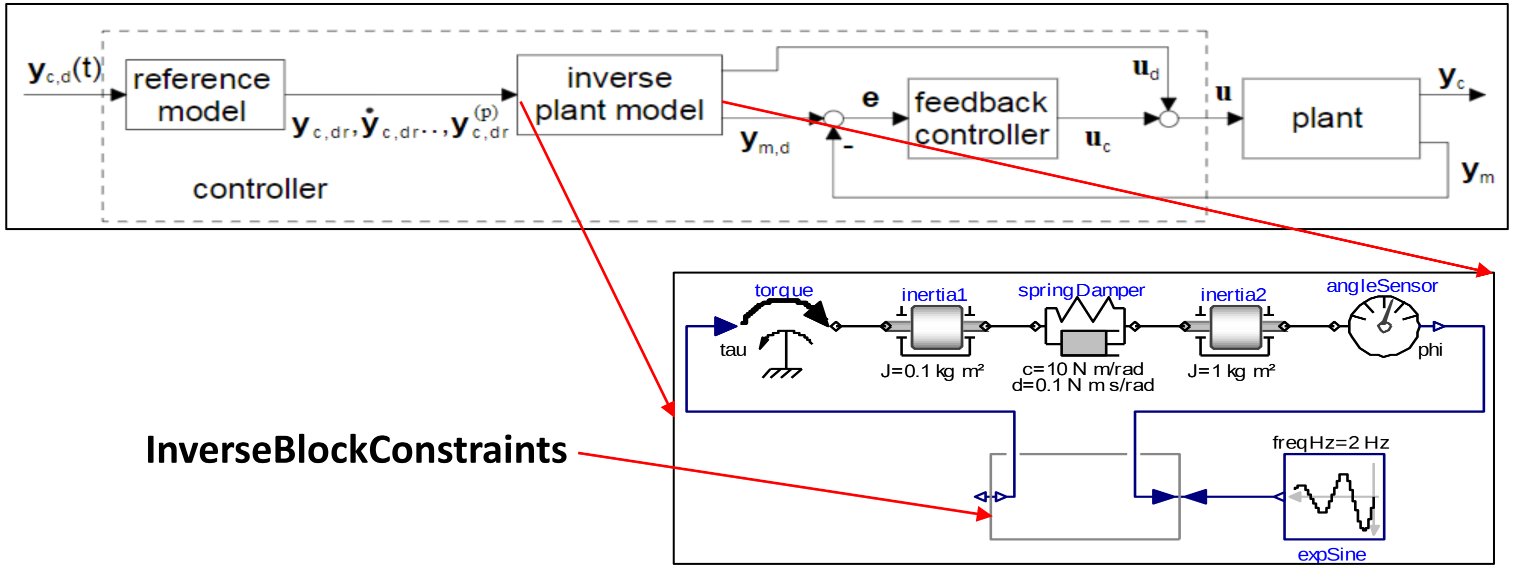
SOLVE DIFFERENT TYPES OF PROBLEMS

- Both the dynamic conditions and the initial conditions are acausal
  - Allows to solve different types of problems: **dynamic/steady-state**, **forward/inverse**
- A dynamic model can be used for steady state initialisation
  - Initialize with v=0 and a=0
- The same model can be used to solve both the forward problem and its inverse (given that there is an inverse solution)
  - Provide an equation for the trajectory and calculate the drive force



INVERSE MODELING

- Model inversion is a powerful feature that has many applications like advanced controller design
- Acausal nature of Modelica models makes it a very good candidate for automatic inversion of models
- Modelica models are automatically rearranged based on boundary condition imposition during translation process in a tool
- Impact's symbolic processing capabilities help the model inversion with very less user effort



STATE SELECTION IN A DYNAMIC SYSTEM

DEGREES OF FREEDOM VS STATE VARIABLES

- In mechanics, the degrees-of-freedom (DOF) are essentially the directions that the system or parts of the system can move without breaking:
  - A body in space has six DOF, three for translational motion, and three for rotational motion.
  - Constraints can remove DOF:
    - A door has one DOF as it can only swing around its hinges
    - A scissor mechanism also only has one DOF
- The minimal set of variables needed to uniquely define the state of the system are called state variables
  - If the set of variables that represent each DOF is known, the state of the system is known.

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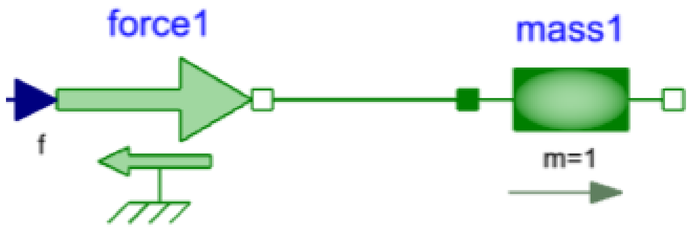
STATE VARIABLES

Consider a 1-D mass



- 2^{nd} order eqn: $m \cdot \ddot{s} = f$
- System of 1^{st} order eqn: $\begin{cases} \dot{s} = v \\ m \cdot \dot{v} = f \end{cases}$

$$\dot{\boldsymbol{x}} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \boldsymbol{x} + \begin{pmatrix} 0 \\ 1/m \end{pmatrix} f \quad \rightarrow \quad \dot{\boldsymbol{x}} = \boldsymbol{Ax} + \boldsymbol{Bu} \quad \rightarrow \quad \dot{\boldsymbol{x}} = \boldsymbol{y}(\boldsymbol{x}, \boldsymbol{u})$$

- Here $\boldsymbol{x}$ is the state vector $\begin{pmatrix} s \\ v \end{pmatrix}$

STATE VARIABLES

$$\dot{x} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \boldsymbol{x} + \begin{pmatrix} 0 \\ 1/m \end{pmatrix} f \quad \rightarrow \quad \dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} \quad \rightarrow \quad \dot{\boldsymbol{x}} = \boldsymbol{y}(\boldsymbol{x}, \boldsymbol{u})$$

- When a numerical integration algorithm is applied (Explicit or Implicit)

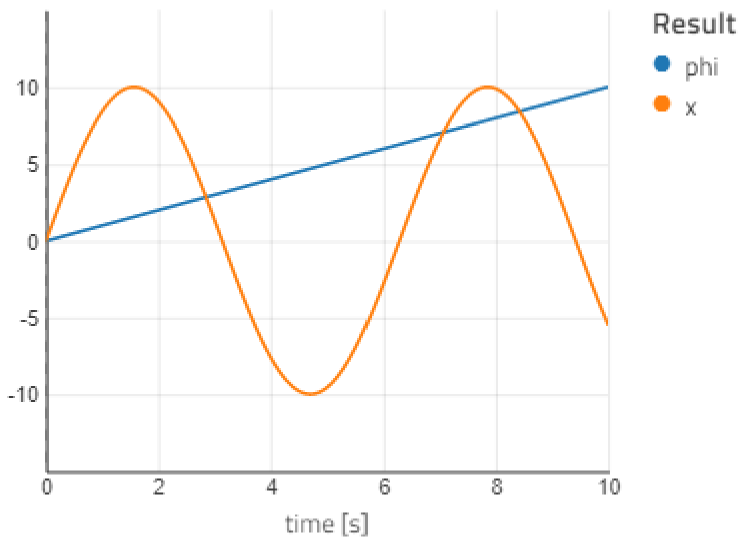
$$\dot{\boldsymbol{x}} = \boldsymbol{y}(\boldsymbol{x}, \boldsymbol{u}) \quad \rightarrow \quad \boldsymbol{X}_{\boldsymbol{n+1}} = \boldsymbol{F}(\boldsymbol{X}_{\boldsymbol{n}}, \boldsymbol{U}_{\boldsymbol{n}}) \quad \textit{or} \quad \boldsymbol{X}_{\boldsymbol{n+1}} = \boldsymbol{F}(\boldsymbol{X}_{\boldsymbol{n+1}}, \boldsymbol{U}_{\boldsymbol{n+1}})$$

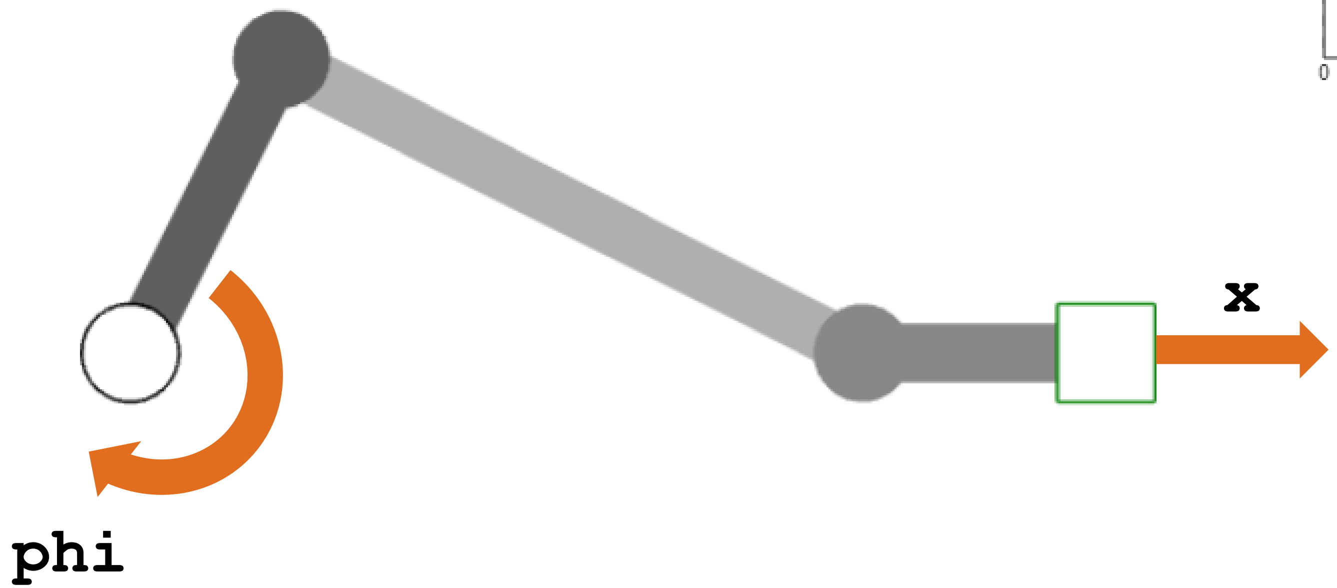
- Where $\boldsymbol{X}$ is the numerical solution
- This implies that you need initial conditions $\boldsymbol{X}_{\boldsymbol{0}}$ for each state

STATE SELECTION

Consider a slider-crank mechanism:

What variable could be a state, x or phi?





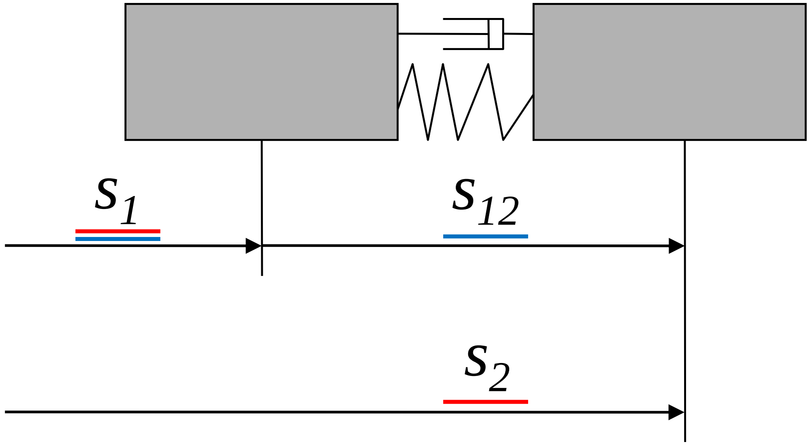
You must compute either:

`x=f(phi);`

`phi=g(x);`

STATE SELECTION

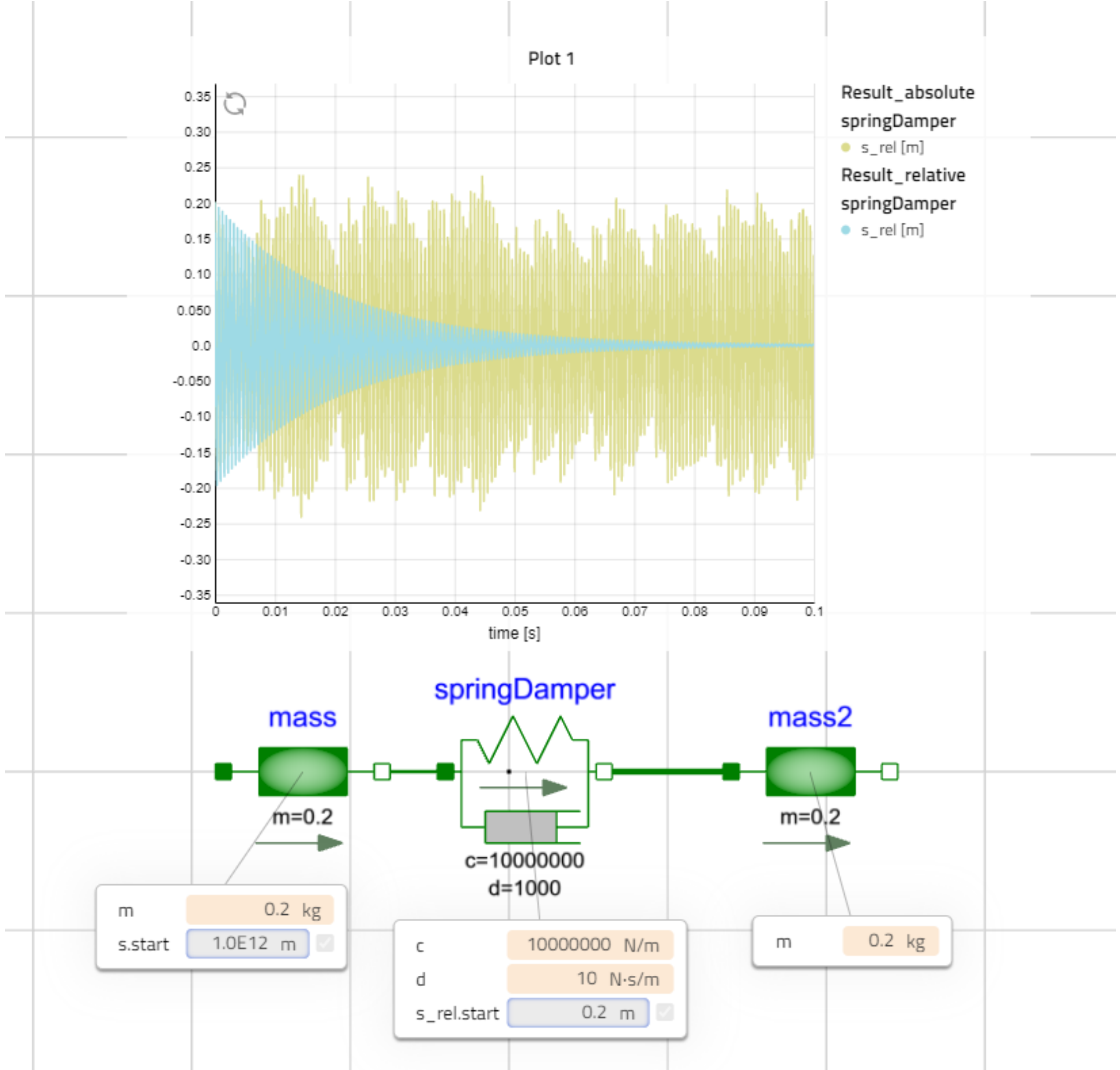
Oscillating masses



$$f = d\, v_{12} + c\, s_{12}$$

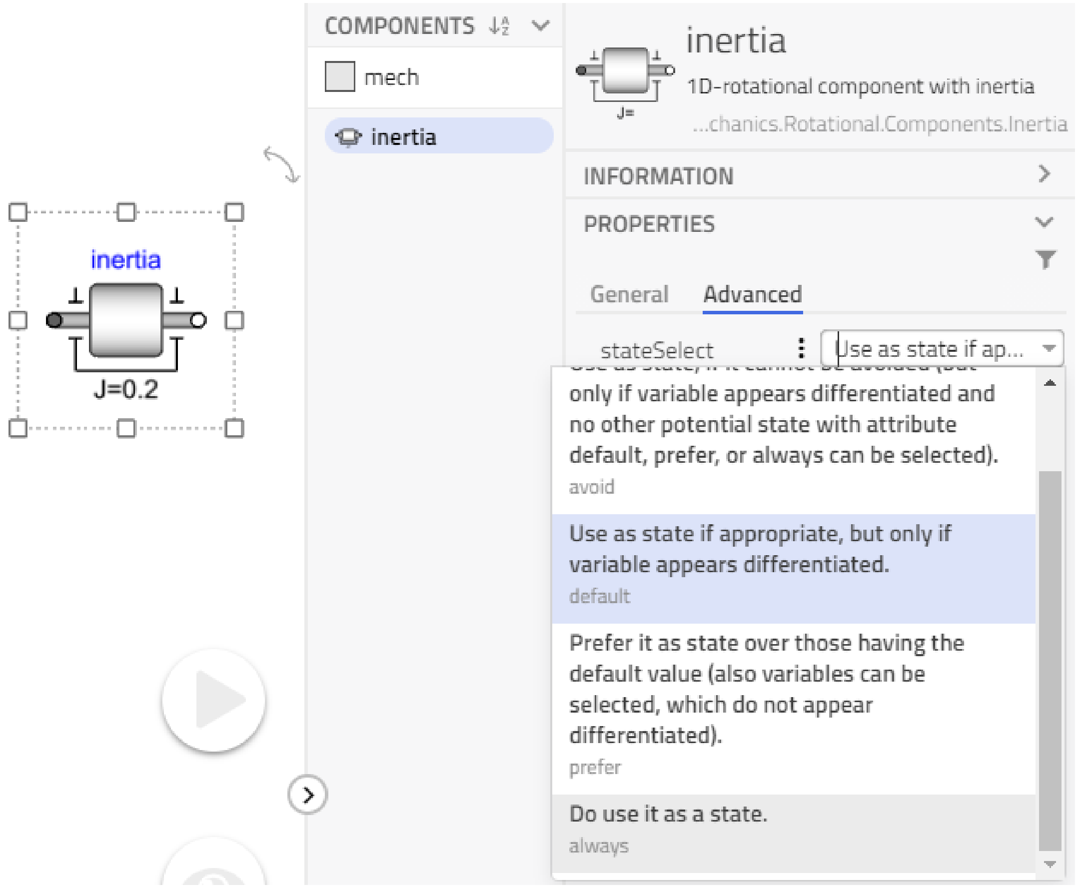
$$s_2 = s_1 + s_{12}$$

Simulation results comparing absolute and relative coordinates



STATE SELECTION

- Can be influenced via attribute of real variables:
  `Real p(stateSelect = StateSelect.`
  - `never`
  - `avoid`
  - `default`
  - `prefer`
  - `always);`



STATE SELECTION - ADVICE

- Avoid state-selection in absolute variables
  - Relative states often numerically better than absolute ones
- In mechanics, rotational DOF is often a better state candidate than translational
- Avoid dynamic state selection
  - Use only to avoid otherwise unavoidable singularities
  - Dynamic state selection can not be combined with real-time solvers!

INITIALIZATION

INITIALIZATION

- A dynamic model describes how the states evolve with time
  - The states are the memory of the model; for example, in mechanical systems, positions and velocities
  - When starting a simulation, the states need to be initialized
- For an ordinary differential equation, ODE, in state space form:

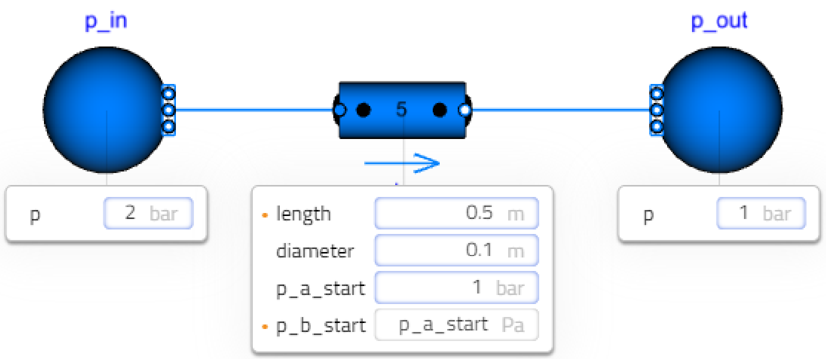
$$d\boldsymbol{x}/dt \;=\; f(\boldsymbol{x}, t)$$

  - the state variables, **x**, are free to be given initial values
  - Initial values can be explicitly set for each state
  - In many cases it is convenient to start at steady state
    - $d\boldsymbol{x}/dt \;=\; 0$ is specified as an initial condition
    - Impact automatically calculates the initial values of **x** by solving:

$$0 \;=\; f(\boldsymbol{x}, t)$$

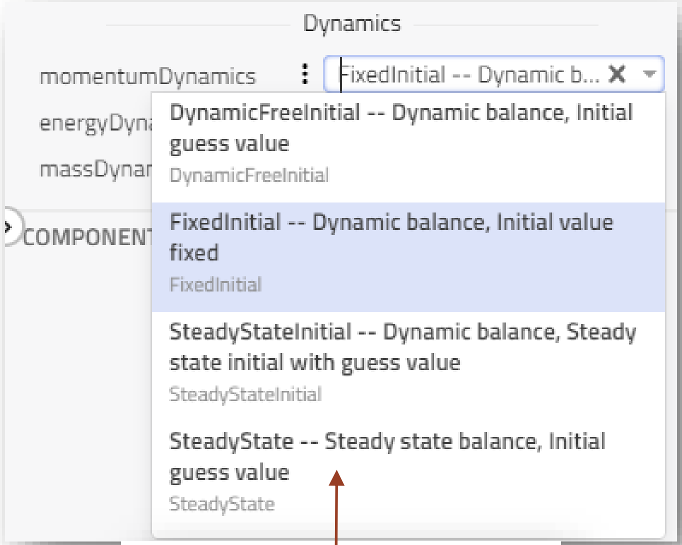
INITIALIZATION

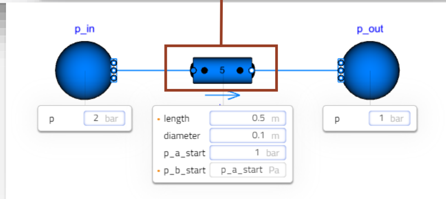
- Most Modelica models have an option to either set an initial value or set an initial guess value.
  - Initial guess values are set when initializing in steady state.
    - Used as a starting point from where the solver tries to find a solution to f(x, t) = 0.
    - Choosing a reasonable guess value is important. Guess values too far away from the actual solution often cause the initialization to fail.
- Example: pipe with two defined pressures at the boundaries
  - Left side 2 bar, right side 1 bar

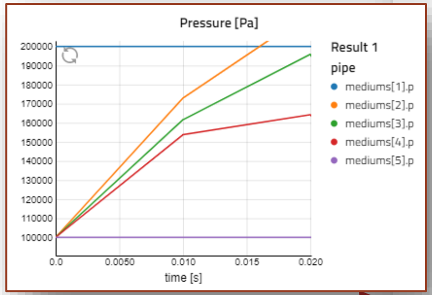


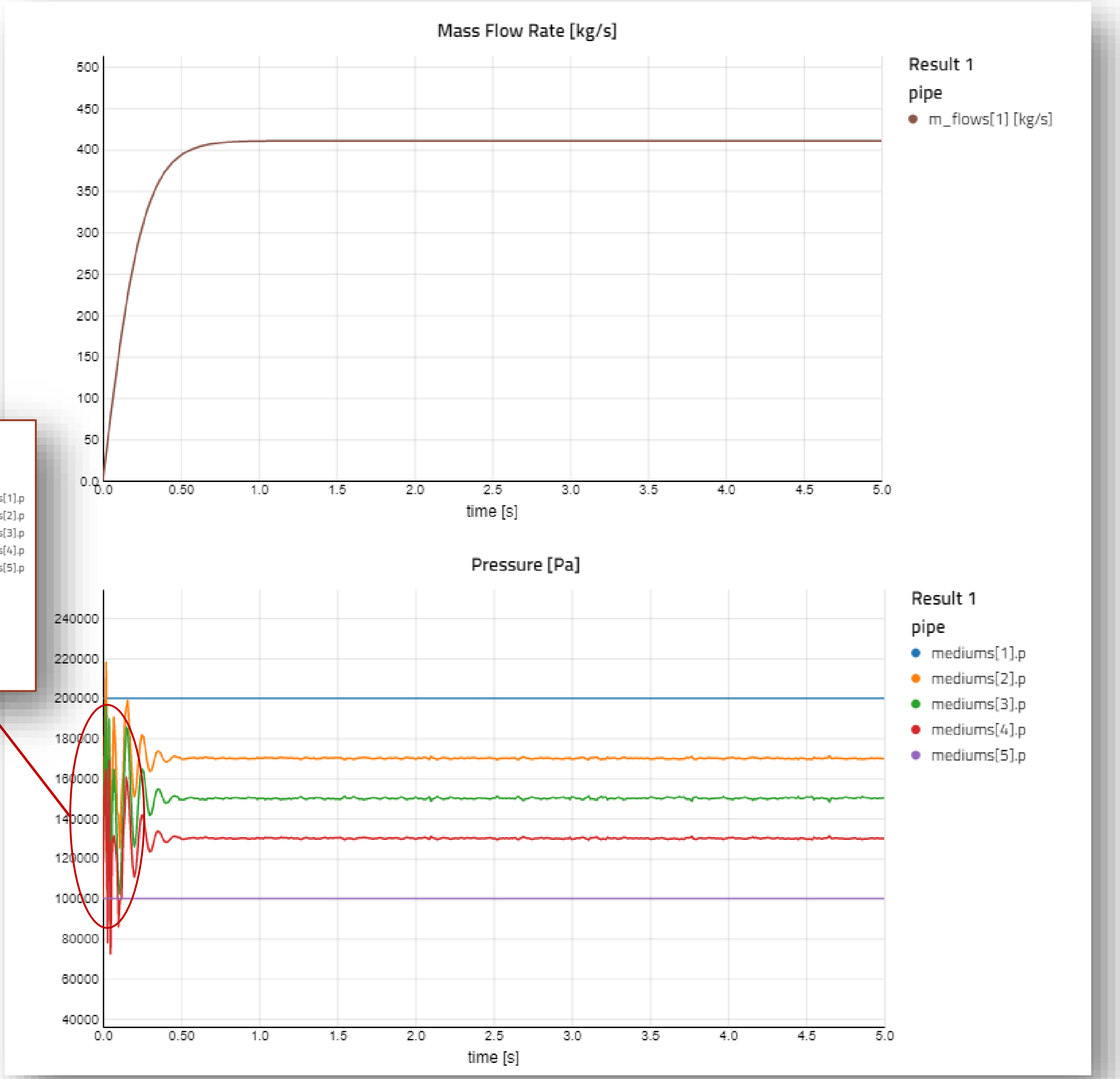
INITIALIZATION

- Setting initial conditions:
  - The simulation starts with the set initial conditions.









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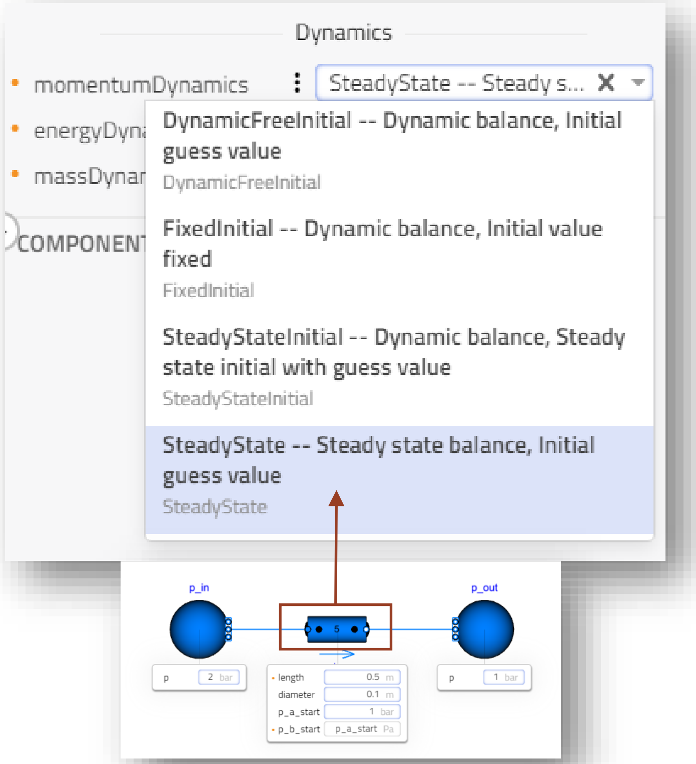
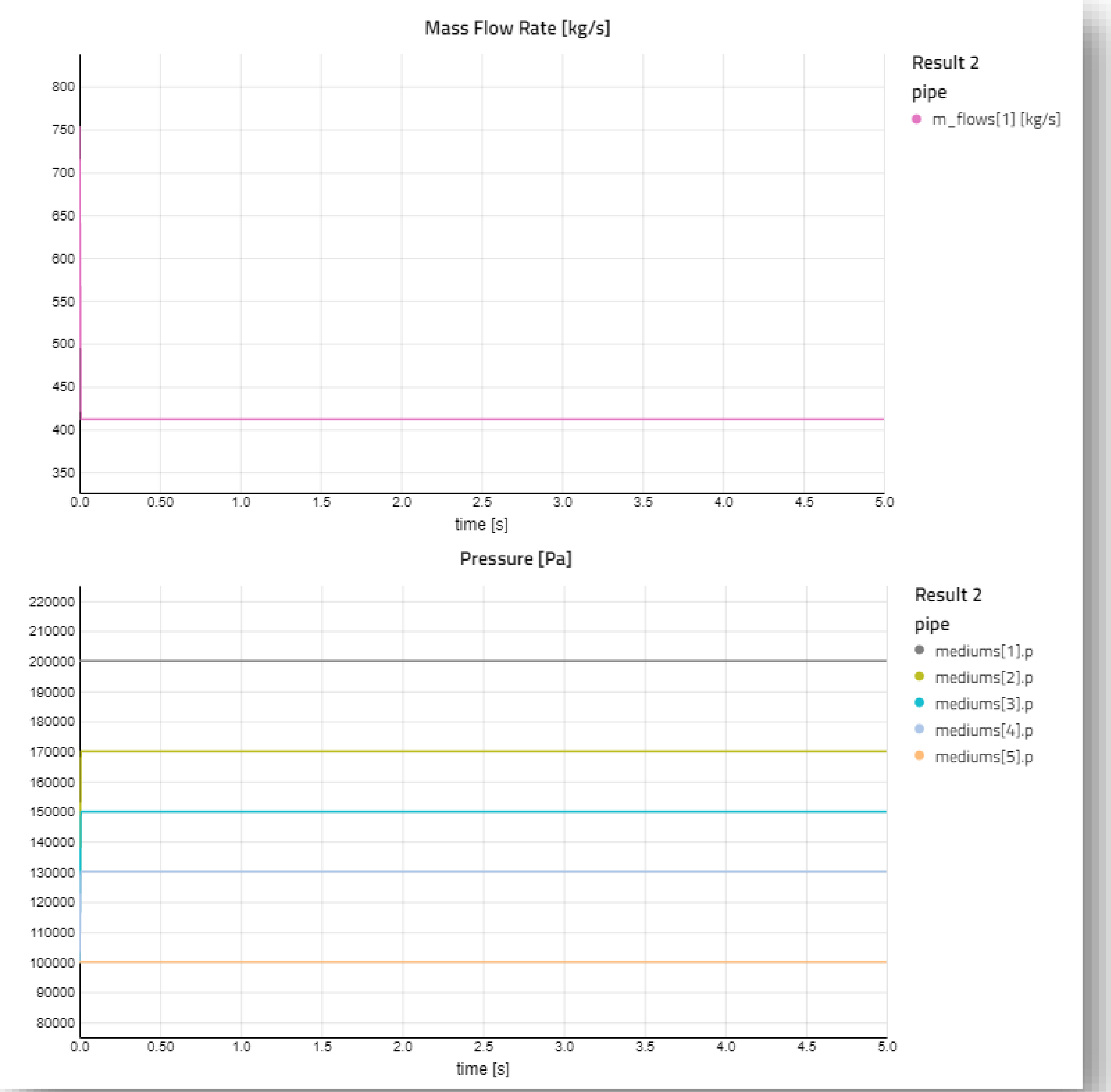
INITIALIZATION

- Steady-state initialization
    - At t=0 all derivatives equal zero





WORKSHOP 2.2

In this workshop you will:

- Investigate a-causal modeling
- Defining boundary conditions
- Model inversion